Bali Learning Center
Quadratic Formula Proof

Let a b and c be any coefficient for quadratic term

$$ax^2 + bx + c$$

Set the equation equal to 0

$$ax^2 + bx + c = 0$$

Subtract by "c"

$$ax^2 + bx = -c$$

Divide equation by "a"

$$x^2 + \frac{bx}{a} = \frac{-c}{a}$$

Complete the Square

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Factor the Perfect Square

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Take the square root of both sides

$$x + \frac{b}{2a} = \pm\sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

Distribute the exponent

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$

Find common denominator

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

Write the expression as a single fraction

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Simplify the denominator in the radical

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Isolate "x"

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Write the expression as a single fraction

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$